PID Controller Improvement with the Modified Tustin Friction Compensation

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Abstract
A PID controller is widely used in the industrial due to a good performance and disturbance rejection. However, the system with a PID controller is still had the un-want characteristics from a friction that exists in an every mechanical system. In this paper, the PID controller with compensation of modified Tustin friction by using the Extended Kalman Filter (EKF) have been introduced and investigated. The results from the computer simulations showed that a PID controller with the friction compensation by using the EKF had the better performances than a system without the friction compensation base on a classic tuning with Ziegler-Nichols. The performances of the system were indicated by the Root-Mean-Square Error (RMS Error) and the Peak Error. The RMS Errors of the system were decrease by 86.05%, 30.20%, and 19.88% for the sinusoidal, triangular, and square input signals, respectively. For the Peak Errors, it was decrease by 73.51% for the sinusoidal input signal, did not changed for the square input signal, and increase by 18.53% for the triangular input signal.

1. Introduction
A proportional–integral–derivative controller (PID controller) is a widely used in industrial control systems. A PID controller calculates an "error" value as the difference between a measured process variable and a desired set-point. The controller attempts to minimize the error by adjusting the process control inputs. The PID parameters used in the calculation must be tuned according to the nature of the system.

Friction is a complicated and not well-understood phenomenon. Its nonlinear behavior consists of unknown parameters, discontinuities and time delays. Because of its complexity and difficulty to model, many design engineers are ineffective at compensating for its effects. As a result, steady state errors, jitter, and limit cycles often degrade the performance of systems. The study of friction often concerns component wear, machine life, and surface chemistry [1]. For the control engineer, on the other hand, the study concerns the compensation for friction, which almost always exists in the system and cannot be avoided.

In this paper, the PID controller with the Modified Tustin friction compensation by using Extended Kalman Filter (EKF) is introduced.

2. Mathematical Model
2.1 Plant model
To investigate the performances of a system, a mechanical plant of a rotating shaft with a friction torque of bearing has been selected for study as show in figure 1.

From the Fig.1, the equation of motion for the plant is given by
\[
\Sigma T = J\alpha \quad (1)
\]
\[
u - T_f = J\dot{y} \quad (2)
\]
or
\[
v = \int \frac{1}{J}(\alpha - T_f) \, dt \quad (3)
\]
where \( u \) is the input torque, \( \nu \) is the angular velocity, \( J \) is the mass moment of inertia, \( \alpha \) is the angular acceleration, and \( T_f \) is the Friction torque of bearing.

From (3), the plant block diagram is showed in figure 2.

The actual friction of the plant is assumed to be the modified Tustin’s friction model, which the mathematical model of the actual friction is given by [1]
\[
T_f = \left( c_0 + c_1 \nu + c_2 \nu^2 \right) \times \text{sign}(\nu) \quad (4)
\]
Where \( v \) is the angular velocity, and \( c_0, c_1, c_2, c_3 \), and \( c_4 \) are the constants.

![Figure 2. The plant block diagram](image)

### 2.2 Control law

From the Figure 1. The transfer function of a mechanical plant of a rotating shaft without a friction is

\[
G_p(s) = \frac{V(s)}{U(s)} = \frac{1}{Js}
\]

(5)

From (5), the system is first order with a pole at the origin. We can use a PID controller to improve the performance of a system. The equation of the control law is given by [2], [4]

\[
u(t) = K_p e(t) + K_1 \int e(t) dt + K_d \frac{de(t)}{dt}
\]

(6)

where \( u \) is a plant Input, \( e \) is an error signal, \( K_p \) is a proportional gain, \( K_i \) is an integral gain, \( K_d \) is a derivative gain.

By classical Ziegler–Nichols tuning method, we have \( K_p = 1.2 \ N.m.rad^{-1} \), \( K_i = 3.139 \ s \), and \( K_d = 0.0796 \ N.m.s.rad^{-1} \) for the design criteria of damping ratio less than 0.7 and time constant less than 1 second with mass moment of inertia, \( J = 0.1 \ kg.m^2 \).

### 3. Compensation and estimation of friction

The closed-loop system block diagram for the plant is shown in Figure 3. The control unit consists of two parts, the PID controller and the EKF. The estimated friction signal, \( \hat{T}_f(k) \), is generated by the EKF and then added to control signal from the PID controller.

![Figure 3. The closed-loop-system block diagram](image)

To estimate the friction by EKF, the discrete equation of the friction is given by [2]

\[
\hat{T}_f(k) = a(k)sgn(\hat{v}(k))
\]

(7)

and

\[
a(k+1) = a(k) + w(k)
\]

(8)

where \( T_f(k) \) is the friction, \( \hat{v}(k) \) is the angular velocity, and \( w(k) \) is a random white noise.

From equation (7) and (8), The discrete block diagram for the plant is shown in Figure 4, where \( T \) is a sampling period, \( z(k) \) is a measurement signal, \( n(k) \) is a measurement noise signal, and \( x_1(k), x_2(k) \) are the system state variables.

![Figure 4. The plant discrete block diagram](image)

From the Figure 4, the system state equation is given by

\[
X(k+1) = f(X(k),u(k)) + G \times w(k)
\]

(9)

and

\[
z(k) = h(X(k)) + n(k)
\]

(10)

Due to the nonlinear of the system in equation (9) and (10), we use the Extended Kalman Filter (EKF) to estimate the state of the system as follow:

The error covariance matrix and the estimated state before measurement is given by

\[
P(k+1) = A((X(k),u(k)) \times P(k) \times A^T((X(k),u(k))) + G \times Q \times G^T
\]

(11)

Also, the estimated state after measurement is given by

\[
\hat{X}(k+1) = f(\hat{X}(k),u(k))
\]

(12)

\[
\hat{X}(k+1) = \hat{X}(k+1) + K(k+1) \cdot (z(k+1) - h(\hat{X}(k+1)))
\]

(13)

where

\[
K(k+1) = P(k+1) \cdot C^T(\hat{X}(k+1)) \cdot [C(\hat{X}(k+1) + P(k+1) \cdot C^T(\hat{X}(k+1)) + R)^{-1}
\]

(14)

For the friction estimation, it’s given by equation (7)

\[
\hat{T}_f(k) = \hat{a}(k)sgn(\hat{v}(k))
\]

(15)

and

\[
\hat{a}(k) = \hat{x}_2(k), \quad \hat{v}(k) = \hat{x}_1(k)
\]

(16)

(17)

### 4. Computer simulation result

The computer simulations are conducted for two cases of compensation, no friction compensation...
(NFC) and with friction compensation (WFC). For NFC, the block EKF in the figure 3 is removed, whereas the block diagram of WFC is as in the figure 3. For each case, the input signals are consist of sinusoidal, triangular, and square signal. Also, the constant $c_0=0.1$, $c_1=0.5$, $c_2=0.1$, $c_3=0.1$, and $c_4=2$ for the modified Tustin’s friction model in equation (4) are use for simulation.

The plots of simulation compared between the input and the output signals with sinusoidal, triangular, and square inputs are shown in figure 5-10 for NFC and WFC respectively.

Figure 5. The response of the system with sine input signal without friction compensation

Figure 6. The response of the system with sine input signal with friction compensation

Figure 7. The response of the system with triangular input signal without friction compensation

Figure 8. The response of the system with triangular input signal with friction compensation

Figure 9. The response of the system with square input signal without friction compensation
input signal, did not changed for the square input signal, and increase by 18.53% for the triangular input signal.

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References

